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Analiza uspeha učenika sedmog razreda na državnim matematičkim takmičenjima u odnosu na oblast i tip zadataka: poređenje učenika iz specijalizovanih i klasičnih odeljenja[1]

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Apstrakt *Matematička takmičenja sve više privlače pažnju istraživača u oblasti matematičkog obrazovanja, budući da donose brojne benefite za učenike – od produbljivanja i proširivanja matematičkih znanja do razvoja kompetencija za rešavanje problema. Imajući u vidu da je učenicima sedmog razreda u Republici Srbiji već dugi niz godina omogućeno pohađanje specijalizovanih odeljenja učenika sa posebnim sposobnostima za matematiku (SM odeljenja), postavlja se pitanje da li njihovo grupisanje u homogena odeljenja utiče na postignuća na državnom takmičenju iz matematike. Cilj ovog rada je da se podrobnije uporede postignuća učenika sedmog razreda SM i klasičnih odeljenja na državnom takmičenju, kao i da se analiziraju zadaci na kojima su učenici dve grupe ostvarili statistički značajno različite rezultate. Analizom uspeha u rešavanju 50 takmičarskih zadataka utvrđeno je da su učenici SM odeljenja ostvarili statistički značajno bolje rezultate u 23 zadatka, dok su učenici klasičnih odeljenja imali značajno bolje rezultate u samo jednom zadatku. Najveće razlike u korist učenika SM odeljenja zabeležene su u zadacima iz teorije brojeva, zatim geometrije i kombinatorike, dok su najmanje razlike uočene u zadacima iz algebre. Takođe, učenici SM odeljenja su u značajno većoj meri bili uspešniji u rešavanju dokaznih zadataka nego učenici klasičnih odeljenja. Detaljna analiza sadržaja zadataka ukazuje da razlike u postignućima mogu biti posledica šireg i temeljnijeg matematičkog obrazovanja kojem su učenici SM odeljenja izloženi, kao i njihove bolje sposobnosti za sintezu različitih matematičkih pojmova i tvrđenja u postupku rešavanja. Rezultati ovog istraživanja mogu poslužiti kao smernice nastavnicima i učenicima u pripremi za buduća matematička takmičenja.*

1 Rad nastao u okviru projekta *Prediktori postignuća učenika 7. i 8. razreda na matematičkim takmičenjima – PREDIMAT 7–8*. finansiranog od strane Centra za naučnoistraživački rad SANU i Univerziteta u Kragujevcu.

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Ključne reči: *matematička takmičenja, tip odeljenja, matematičke oblasti, tipovi zadataka, postignuća učenika.*

Uvod

Takmičenja iz matematike učenika osnovnih škola imaju dugu tradiciju u Republici Srbiji. Od prvih takmičenja šezdesetih godina prošlog veka i prvog republičkog takmičenja 1967. godine (Mićić i Kadelburg, 2022) sve do danas, postala su najmasovnija takmičenja, na kojima svake godine između 60000 i 70000 učenika od 4. do 8. razreda započinje takmičarski ciklus (Vulović i sar., 2023).

Državno takmičenje iz matematike je četvrti nivo takmičenja, na kojem učestvuju učenici od 6. do 8. razreda osnovne škole. Iako do sada nije rađeno mnogo istraživanja koja se bave konkretnim postignućima učenika na ovim takmičenjima, dostupna istraživanja pokazuju da učenici koji pohađaju specijalizovana odeljenja učenika sa posebnim sposobnostima za matematiku (SM odeljenja) postižu statistički značajno bolja postignuća u odnosu na učenike koji pohađaju klasična odeljenja. Vulović i sar. (Vulović i sar., 2024) empirijski su utvrdili da su učenici SM odeljenja u poslednjih 18 godina ostvarili statistički značajno bolje rezultate čak 11 puta.

Ovim radom želimo da ispitamo postignuća učenika sedmog razreda osnovne škole na državnim takmičenjima iz matematike u odnosu na oblast i tip zadataka, u zavisnosti od toga da li učenici pohađaju klasična ili SM odeljenja u periodu od 2015. do 2024. godine. Razlog tome nalazimo u činjenici da su zadaci koji su obuhvaćeni programima takmičenja, idući od školskog ka međunarodnom nivou matematičkih takmičenja, sve manje zasnovani na školskom kurikulumu. Analizom zadataka koje su učenici sa najboljim rezultatima na državnim takmičenjima u posmatranom periodu dominantno uspešnije rešavali, želimo da pružimo doprinos u identifikaciji faktora koji utiču na uspeh učenika na nacionalim matematičkim takmičenjima.

Teorijski okvir istraživanja

Jedna od funkcija matematičkih takmičenja jeste buđenje interesovanja za dublje bavljenje matematikom i razvoj kompetitivnosti među učenicima. Ona pružaju podršku učenicima budući da učenici, tokom rešavanja zadataka, demonstriraju svoje visoke matematičke sposobnosti do čijeg je formiranja došlo tokom procesa pripreme za matematička takmičenja (Keldibekova, 2020). Kenderov navodi da rešavanje složenih matematičkih zadataka omogućava bolje znanje, ali i razvija veštine učenika za prevazilaženje realnih problemskih situacija (Kenderov, 2022).

Lijedal razlikuje „učionice u kojima se misli“ i „učionice u kojima se uči bez razmišljanja“: prvi tip „učionica“ odnosi se na podsticajno školsko okruženje u kome učenici aktivno razmišljaju prilikom učenja, a drugi tip se odnosi na školsko okruženje u kome učenici tek oponašaju razmišljanje (Liljedahl, 2019). Upravo matematička takmičenja predstavljaju sredinu u kojoj se afirmiše razmišljanje učenika kao važno iskustvo za bavljenje matematikom na višem nivou. Razmatrajući kompetencije učenika potrebne za bavljenje matematikom na višem nivou, Falk de Losada i Tejlor ističu da rešavanje takmičarskih zadataka dovodi do razvijanja kompetencija učenika za rešavanje problema i boljeg usvajanja i korišćenja matematičke terminologije (Falk de Losada & Taylor, 2022). Dok školska matematika prati tradicionalnu strukturu izlaganja sadržaja, matematička takmičenja integrišu date sadržaje sa načinima razmišljanja i strategijama za rešavanje novih problema i izazova, proširujući um učenika i upoznajući ih sa načinom rezonovanja profesionalnih matematičara (Nieto-Said & Sanchez-Lamoneda, 2022).

Zadaci na takmičenjima pripadaju oblastima algebre, geometrije, teorije brojeva i kombinatorike, ali najčešće istovremeno uključuju dve ili više oblasti. Ključ u rešavanju takvih zadataka jeste u domišljatosti, kreativnosti i veštini rešavanja problema. Istraživanja pokazuju da učesnici takmičenja ostvaruju različit uspeh u zavisnosti od oblasti koja dominira u zadatku. Često su uspešniji u rešavanju zadataka iz algebre, kombinatorike i teorije brojeva, u odnosu na zadatke iz geometrije (Facciaroni et al., 2023; Keldibekova, 2020; Špijunović & Maričić, 2013; Vulović i sar., 2024).

Faćaroni i saradnici su na uzorku od 29592 učenika devetog razreda, koji su od 2018. do 2020. godine učestvovali na okružnom nivou takmičenja u Italiji, ispitivali razlike u postignućima učenika u odnosu na matematičku oblast zadataka (Facciaroni et al., 2023). Fokus istraživanja bio je na geometriji i poteškoćama koje učenici imaju u rešavanju ovih zadataka i u izvođenju dokaza. Svake godine, prvih četrnaest od ukupno sedamnaest zadataka na takmičenju nije zahtevalo izvođenje dokaza, ali je bilo neophodno da učenici poznaju teoreme i postupke radi uspešnog rešavanja tih zadataka. Nasuprot tome, poslednja tri zadatka (po jedan iz teorije brojeva, kombinatorike i geometrije) zahtevala su izvođenje dokaza, ali i nosila 40% ukupnog broja bodova, što je impliciralo da pravilno rešavanje upravo tih triju zadataka determiniše koji će učenici nastaviti takmičenje na nacionalnom nivou. Utvrđeno je da postoji razlika u uspešnosti učenika u zavisnosti od oblasti, premda veliki procenat učenika nije ni pokušao da uradi dokazni zadatak iz geometrije. U sve tri godine obuhvaćene istraživanjem, prosečan broj poena osvojenih na dokaznom geometrijskom zadatku bio je znatno niži od prosečnog broja poena druga dva dokazna zadatka. U okviru istog istraživanja, autori su 2020. godine sproveli i anketiranje 486 učesnika okružnog takmičenja u okrugu Rim. Prilikom anketiranja, učenici su odgovorili da se u proseku podjednako prijatno osećaju prilikom rešavanja zadataka iz svake oblasti. Iako nisu bile statistički značajne, razlike u srednjim vrednostima pokazuju da učenici geometriju ipak percipiraju kao najtežu od navedenih oblasti, dok im je teorija brojeva najlakša. Razlog za ovako male razlike u srednjim vrednostima stavova učenika autori pronalaze u činjenici da je skoro trećina učenika odgovorila da su im takmičarski zadaci iz geometrije poznati sa redovne nastave (bilo po složenosti, sadržaju ili strukturi), dok je u slučaju ostale tri oblasti u proseku svaki peti učenik odgovorio da su takmičarski zadaci slični onima koje su rešavali na časovima. To potvrđuje činjenicu da, iako poznaju osnovna pravila geometrije, kojima ovladavaju na časovima redovne nastave, i učenici koji ostvaruju iznadprosečna postignuća imaju poteškoća sa rešavanjem takmičarskih zadataka iz geometrije (Sal'kov et al., 2017).

Iako se mogu rešiti primenom elementarnih metoda, zadaci na matematičkim takmičenjima mogu biti veoma izazovni za rešavanje. Za razliku od konvencionalnih zadataka iz školske matematike, koji obično zahtevaju reprodukciju poznatih procedura, zadaci na takmičenju su inkluzivni, budući da uzimaju u obzir različite situacije, nagađanja, ispitivanje pretpostavki ipostavljanje novih pitanja (Leikin, 2007), pri čemu se njihovo rešenje smatra potpunim ako i samo ako učenik pruži potpun dokaz onoga što tvrdi. Falk de Losada i Tejlor navode da učenici na takmičenjima, slično profesionalnim matematičarima, proces dokazivanja najčešće započinju opisivanjem onoga što se želi dokazati (predlaganjem argumenta), a zatim nastavljaju sa izvođenjem dokaza (Falk de Losada & Taylor, 2022). Tom prilikom, učenici koriste jedan od četiri tipa dokaza (Balacheff, 1988, prema: Laamena et al., 2018): naivni empirizam, kojim se nakon provere posebnih slučajeva potvrđuje istinitost argumenta; ključni eksperiment, koji se koristi za izbor između dve prethodno formulisane hipoteze; generički primer, kojim se objašnjava zašto je argument istinit uz navođenje specifičnih osobina i struktura; i misaoni eksperiment, kojim se argument uopštava i diferencira od posebnih slučajeva.

Uticaj grupisanja učenika na njihova postignuća jeste tema koja je razmatrana u literaturi, ali je u manjoj meri ispitivan uticaj homogenog grupisanja na postignuća učenika na takmičenjima. Glavni cilj ovakvog tipa grupisanja jeste da se zadovolje interesovanja i specifične obrazovne potrebe učenika koji imaju posebne sposobnosti za matematiku, a koji najčešće učestvuju na matematičkim takmičenjima. Karp navodi da nastavu u školama u kojima su matematički daroviti učenici grupisani u posebno odeljenje karakteriše postavljanje problema, rešavanje problema, traženje originalnih metoda rešavanja problema i pronalaženje veza između novih i starih problema (Karp, 2009). Kulik i Kulik su sproveli 90 studija obuhvaćenih metaanalizom i među njima izdvojili 25 istraživanja čiji je predmet bilo ispitivanje efekata nastave u odeljenjima u kojima su grupisani učenici sa posebnim sposobnostima (Kulik & Kulik, 1987). Rezultati 19 studija pokazali su da daroviti učenici postižu bolje rezultate kada su grupisani u homogena odeljenja, dok su u 11 istraživanja te razlike statistički značajne. Isti autori takođe navode da grupisanje darovitih učenika u homogena odeljenja ne daje željene efekte ukoliko izostane odgovarajuća transformacija kurikuluma kojom bi se zadovoljile njihove obrazovne potrebe. Ove nalaze potvrđuju i rezultati studije Stinbergen-Hu i saradnika, koja je obuhvatila sto godina istraživanja efekata koje grupisanje učenika ostavlja na njihova akademska postignuća (Steenbergen-Hu et al., 2016).

Pozitivan uticaj prelaska učenika iz klasičnih odeljenja u gimnazije sa organizovanom nastavom za učenike osnovne škole posebno nadarene za matematiku uočen je i u studiji Asmata (Asmat, 2021). S obzirom na mogućnost da učenici posle završenog petog ili osmog razreda upišu ovakva odeljenja, jedan od ciljeva istraživanja Asmata bio je i da se utvrdi u kojoj meri izbor škole i vrsta odeljenja utiče na ostvarivanje postignuća učenika na takmičenjima. Pokazalo se da učenici koji su nastavu pohađali u gimnazijama ostvaruju prosečno za 6% viša postignuća od učenika iz heterogenih odeljenja. Ovakav podatak potvrđuje prednosti načina rada koji se sprovodi u odeljenjima homogenog tipa, u kojima je akcenat na obogaćenom programu nastave matematike (Kim, 2016; Rochayati & Rochayani, 2024).

Sriraman navodi da su složeni matematički zadaci odličan način da se učenicima pruži šansa da razvijaju matematičke procese višeg reda (pre svega reprezentaciju, apstrakciju i generalizaciju) (Sriraman, 2003). U studiji koju je sproveo, Sriraman je grupi od devet učenika devetog razreda dao zadatak da reše pet nerutinskih kombinatornih problema, pri čemu je generalizacija koju je trebalo upotrebiti za rešavanje pet problema data u obliku Dirihleovog principa. Četiri učenika koji su prethodno identifikovani kao matematički daroviti bili su uspešni u otkrivanju i formulaciji opštosti, koju je trebalo upotrebiti za rešavanje tih pet zadataka, dok, sa druge strane, pet učenika koji nisu posedovali posebne sposobnosti za matematiku nisu uspeli da prepoznaju generalizaciju skrivenu u problemu (Sriraman, 2003).

Sama suština grupisanja matematički darovitih učenika nije izdvajanje od vršnjaka prosečnih sposobnosti, već spajanje sa onima koji dele ista interesovanja. Kada su učenici grupisani u ista odeljenja, imaju priliku da ih vršnjaci istih sposobnosti stimulišu da brže i efikasnije obrađuju složenije sadržaje (Neihart, 2007; Swiatek & Lupkowski-Shoplik, 2003), jer nastavu matematike u ovakvim odeljenjima odlikuju strategije akceleracije, obogaćivanja i produbljivanja.

U svom istraživanju su Vulović, Milenković i Milikić (Vulović i sar., 2024) analizirali zastupljenost i postignuća na državnim takmičenjima iz matematike u Srbiji koja su u periodu tokom osamnaest godina ostvarivali učenici sedmog i osmog razreda koji pohađaju SM odeljenja pri gimnazijama i učenici iz heterogenih odeljenja. Autori ističu da su i u sedmom i u osmom razredu prosečne vrednosti broja bodova koje su ostvarili učenici SM odeljenja svake godine bile više u odnosu na učenike iz klasičnih odeljenja. Prosečni rezultati ukazuju da su učenici sedmog razreda iz SM odeljenja za 39% imali viša postignuća u odnosu na učenike iz klasičnih odeljenja, dok je uspeh učenika osmog razreda bolji za 32% od uspeha učenika iz klasičnih odeljenja.

Metodologija istraživanja

Zakon u Republici Srbiji prepoznaje mogućnost grupisanja učenika darovitih za matematiku u posebna odeljenja. Po završetku šestog razreda osnovne škole, učenici imaju mogućnost prelaska u gimnazije koje organizuju nastavu za učenike sa posebnim sposobnostima za matematiku (Pravilnik o nastavnom planu i programu za učenike sedmog i osmog razreda osnovnog obrazovanja i vaspitanja obdarene za matematiku, 2019). Prilikom upisa, učenici polažu poseban kvalifikacioni ispit na kome se vrednuju sledeće stavke: rezultati na testu sposobnosti iz matematike, opšti uspeh u petom i šestom razredu, ocene iz matematike u ovim razredima i učešće na državnom takmičenju iz matematike u šestom razredu.

Takmičenje iz matematike na državnom nivou održava se svake godine u maju, a pravo učešća na njemu ima približno po sto učenika šestog, sedmog i osmog razreda osnovne škole sa najboljim postignućima na okružnim takmičenjima u toku te školske godine.

Cilj ovog istraživanja je analiza postignuća učenika sedmog razreda SM i klasičnih odeljenja na državnim takmičenjima iz matematike u odnosu na matematičke oblasti kojim zadaci sa takmičenja pripadaju i u odnosu na tip zadatka. U skladu sa ciljem, ispitivane su razlike u postignućima učenika klasičnih i SM odeljenja u rešavanju tih zadataka, kao i analiza sadržaja datih zadataka. Iz cilja proističu zadaci istraživanja kojima treba utvrditi:

1) odnos broja zadataka u kojima su učenici SM odeljenja ostvarili bolje rezultate od učenika iz klasičnih odeljenja, broja zadataka u kojima su ove dve grupe učenika postigle jednake rezultate i broja zadataka u kojima su učenici iz klasičnih odeljenja ostvarili bolje rezultate od učenika iz SM odeljenja;
2) odnos broja zadataka u kontekstu postignuća učenika SM i klasičnih odeljenja u zavisnosti od matematičke oblasti kojoj zadaci pripadaju;
3) odnos broja zadataka u kontekstu postignuća učenika SM i klasičnih odeljenja u zavisnosti od toga da li su zadaci računski ili dokazni.

U radu je vršena sekundarna analiza postignuća učenika 7. razreda na državnim takmičenjima iz matematike, u periodu od 2015. do 2024. godine. Učenici su na takmičenjima rešavali po pet zadataka koje je sastavila Državna komisija za takmičenje učenika osnovnih škola, a za čiju su izradu imali 180 minuta. Svi zadaci su javno dostupni na zvaničnom sajtu Društva matematičara Srbije. Državno takmičenje se svake godine održava uživo, kao i na svim prethodnim nivoima, u školskim uslovima, na osnovu Pravilnika o takmičenjima. Sadržaji zadataka su definisani Programom takmičenja koji donosi Društvo matematičara Srbije na početku takmičarskog ciklusa. Pravilnik i Program takmičenja su javno dostupni. Ocenjivanje zadataka se vrši na osnovu precizno definisane skale i uputstva za pregledanje (zadaci se boduju parcijalno). Svaki zadatak se boduje maksimalno sa 20 bodova.

Rezultati dobijeni na državnim takmičenjima svake godine se objedinjuju u jedinstvenu bazu koja je korišćena u ovom istraživanju. Uzorak istraživanja čine svi učenici sedmog razreda koji su učestvovali na državnim takmičenjima iz matematike u periodu od 2015. do 2024. godine, čiji ukupan broj iznosi 1014. Struktura populacije u odnosu na tip odeljenja koje učenici pohađaju data je u Tabeli 1.

Tabela 1
Struktura populacije istraživanja

Godina	Klasično odeljenje		SM odeljenje	
	F	%	*F*	%
2015.	64	66,67%	32	33,33%
2016.	50	50,00%	50	50,00%
2017.	57	58,16%	41	41,84%
2018.	53	52,48%	48	47,52%
2019.	60	62,50%	36	37,50%

Godina	Klasično odeljenje		SM odeljenje	
	F	%	*F*	%
2020.	60	61,22%	38	38,78%
2021.	50	50,00%	50	50,00%
2022.	61	54,46%	51	45,54%
2023.	64	61,54%	40	38,46%
2024.	67	61,47%	42	38,53%
Ukupno	586	57,79%	428	42,21%

Dobijeni podaci obrađeni su u softverskom paketu SPSS 20. Od statističkih mera i postupaka korišćeni su: frekvencije, procenti, aritmetička sredina, statistički test Šapiro-Vilk (engl. Shapiro-Wilk) za određivanje normalnosti raspodele numeričkih podataka, Studentov t-test za nezavisne grupe i Man-Vitnijev (engl. Mann-Whitney) za poređenje raspodele podataka u dve grupe.

Rezultati istraživanja i diskusija

U skladu sa ciljem istraživanja, analizirali smo da li postoje statistički značajne razlike u postignućima učenika koji pohađaju SM i klasična odeljenja, u rešavanju zadataka za sedmi razred, na državnim takmičenjima iz matematike, za svaki zadatak pojedinačno, u prethodnih deset godina (ukupno 50 zadataka). Za svaki zadatak je u obe grupe učenika ispitano da li broj poena koji su učenici ostvarili ima normalnu raspodelu. U slučajevima kada su obe grupe imale normalnu raspodelu, Studentovim t-testom nezavisnih grupa upoređivane su aritmetičke sredine broja ostvarenih poena, dok su u suprotnim slučajevima Man-Vitnijevim testom upoređene raspodele broja poena koje su učenici dveju grupa ostvarili. Rezultati su prikazani u Tabeli 2. Za svaki zadatak su navedene aritmetičke sredine broja poena koje su učenici dve grupe ostvarili, dok su rezultati Man-Vitnijevog testa navedeni u slučajevima kada su ustanovljene statistički značajne razlike. Ni u jednom zadatku u kome su razlike ispitivane Studentovim t-testom nisu ustanovljene statistički značajne razlike u rezultatima učenika.

Tabela 2

Postignuća takmičara sedmog razreda, u odnosu na vrstu odeljenja, po zadacima, u periodu od 2015. do 2024. godine

Godina	Redni broj zadatka									
	1.		2.		3.		4.		5.	
	Klasično	SM	Klasično	SM	Klasično	SM	Klasično	SM	Klasično	SM
2015.	5,72	8,97	**2,73**	**5,56**	2,39	4,09	6,42	9,13	1,31	3,50
			***U*=798.0 *Z*=-3.100 *p*=.002**							
2016.	7,90	8,38	9,86	12,72	**3,44**	**8,78**	**12,00**	**15,98**	3,64	4,56
					***U*=914.0 *Z*=-2.414 *p*=.016**		***U*=1009.5 *Z*=-1.994 *p*=.046**			
2017.	**7,00**	**11,56**	**4,30**	**9,85**	**4,65**	**9,90**	**3,21**	**8,02**	**7,18**	**11,32**
	***U*=798.0 *Z*=-2.766 *p*=.006**		***U*=644.5 *Z*=-3.946 *p*=.000**		***U*=782.5 *Z*=-2.833 *p*=.005**		***U*=861.5 *Z*=-2.349 *p*=.019**		***U*=849.0 *Z*=-2.371 *p*=.018**	
2018.	*17,08*	*16,96*	**8,58**	**12,54**	2,17	2,35	3,02	4,73	**6,57**	**13,42**
			***U*=990.0 *Z*=-2.071 *p*=.038**						***U*=742.0 *Z*=-3.800 *p*=.000**	
2019.	10,32	13,39	**6,75**	**12,17**	10,00	12,06	6,90	8,06	**7,03**	**3,72**
			***U*=664.5 *Z*=-3.178 *p*=.001**						***U*=862.0 *Z*=-1.962 *p*=.050**	
2020.	9,62	13,08	**8,55**	**13,95**	**10,50**	**18,71**	**2,67**	**6,58**	**4,40**	**9,76**
			***U*=745.0 *Z*=-3.056 *p*=.002**		***U*=591.0 *Z*=-4.545 *p*=.000**		***U*=829.5 *Z*=-2.394 *p*=.017**		***U*=690.0 *Z*=-3.491 *p*=.000**	
2021.	6,00	7,60	11,06	12,18	**11,10**	**15,62**	14,22	15,54	*2,00*	*1,20*
					***U*=865.0 *Z*=-2.987 *p*=.003**					
2022.	**9,18**	**12,65**	**2,26**	**7,94**	4,25	4,90	**8,08**	**12,84**	**0,84**	**5,35**
	***U*=1193.5 *Z*=-2.306 *p*=.021**		***U*=866.5 *Z*=-4.523 *p*=.000**				***U*=1063.5 *Z*=-2.960 *p*=.003**		***U*=1146.5 *Z*=-3.147 *p*=.002**	
2023.	6,53	8,75	4,48	7,03	10,08	12,15	4,84	5,50	1,39	3,57
2024.	*18,33*	*17,83*	**10,21**	**15,14**	9,76	11,57	**2,96**	**7,07**	**11,30**	**14,38**
			***U*=931.0 *Z*=-3.064 *p*=.002**				***U*=1011.5 *Z*=-2.781 *p*=.005**		***U*=1080.0 *Z*=-2.095 *p*=.036**	

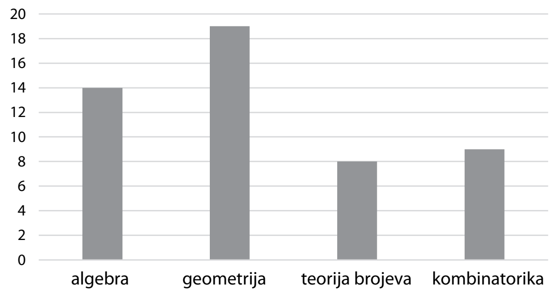
Od 50 posmatranih zadataka, učenici SM odeljenja postigli su statistički značajno bolje rezultate u 23 zadatka, u 26 zadataka razlike nisu bile statistički značajne, dok su učenici klasičnih odeljenja u jednom zadatku postigli statistički značajno bolje rezultate.

Uspeh učenika u zavisnosti od oblasti kojima pripadaju zadaci

Već je napomenuto da zadaci pripadaju oblastima algebre, geometrije, teorije brojeva i kombinatorike, a da su za rešavanje određenog broja zadataka potrebna znanja i umenja iz dve ili tri oblasti. Svaki zadatak svrstali smo u jednu oblast koja je dominantna u njegovom rešavanju sudeći po sadržajima i postupcima. Na osnovu Grafikon 1, uočavamo da je u posmatranom periodu najveći broj zadataka na državnim takmičenjima bio iz geometrije, zatim iz algebre, pa kombinatorike, a najmanji broj iz oblasti teorija brojeva.

Grafikon 1

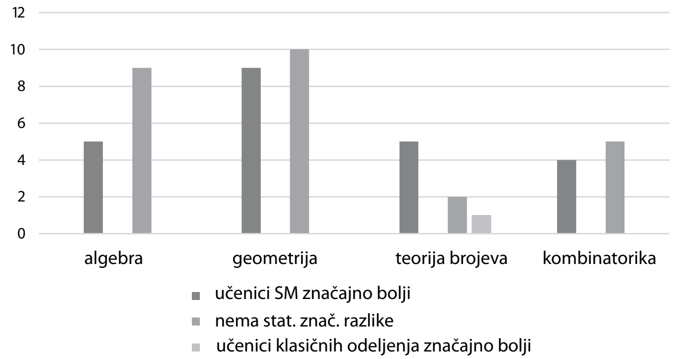
Broj zadataka na državnim takmičenjima učenika 7. razreda od 2015. do 2024. godine, po oblastima



Ako uporedimo uspeh po zadacima učenika SM i klasičnih odeljenja, u odnosu na oblasti kojima zadaci pripadaju, dobijamo rezultate prikazane na Grafikonu 2.

Grafikon 2

Postignuća učenika 7. razreda iz SM i klasičnih odeljenja u odnosu na oblast iz koje su zadaci na takmičenju u periodu od 2015. do 2024. godine



Uočavamo da je razlika u postignućima u korist učenika SM odeljenja najveća na zadacima iz teorije brojeva. Od 8 zadataka iz ove oblasti, učenici SM odeljenja su u 5 zadataka imali statistički značajno bolji rezultat. Analizom sadržaja ovih zadataka uočavamo da svi sadrže delove koji se mogu rešiti poznavanjem sadržaja sa redovne nastave, npr. rastavljanja broja na proste činioce ili deljivosti i ostatka pri deljenju prirodnim brojem, ali da je za uspešno sprovođenje čitavog rešenja obično neophodno i dobro poznavanje sadržaja dodatne nastave, npr. dešifrovanje brojeva u dekadnom zapisu u skladu sa datim uslovima, primena Dirihleovog (kombinatornog) principa (na deljivost prirodnim brojem). Stoga je npr. primena kongruencija za određivanje ostatka pri deljenju, kao i specijalno za određivanje poslednje cifre višecifrenog broja (kao specijalan slučaj deljenja sa 10), nešto sa čime se prosečan učenik osnovne škole ne susreće, osim ako nastavnik tome ne posveti posebnu pažnju na dodatnoj nastavi. S druge strane, veće su šanse da će učenici sedmog razreda SM odeljenja imati priliku da nauče osobine kongruencija jer se one i javljaju u njihovom programu rada. Pored toga, Dirihleov princip se takođe obrađuje isključivo na dodatnoj nastavi matematike (i naglašen je u programu takmičenja). Napominjemo da se zadaci u kojima se primenjuje Dirihleov princip svrstavaju u kombinatorne, ali da je jedan konkretan zadatak (iz 2018. godine) klasifikovan u teoriji brojeva, budući da je u rešavanju akcenat na primeni osobina deljivosti.

Na geometrijskim zadacima su učenici SM odeljenja na 9 zadataka ostvarili statistički značajno bolje rezultate, dok razlika u rezultatima kod 10 zadataka nije postojala. Sadržaji zastupljeni u ovim zadacima su usvajani pre sedmog razreda (osobine kvadrata i pravougaonika, uglova i trouglova, podudarnost trouglova, površina trouglova i četvorouglova, osna simetrija, simetrala duži, simetrala ugla, opisana i upisana kružnica u trougao, itd), a od sadržaja sedmog razreda zastupljeni su: Pitagorina teorema sa primenama na trougao, četvorougao i mnogougao, krug, periferijski i centralni ugao. Posebno treba naglasiti da se u svim zadacima očekivalo poznavanje većeg broja definicija, tvrđenja i osobina iz različitih tema, kao i njihovo sintetisanje u rešenje zadatka. Iz tog razloga je bilo zadataka u kojima je trebalo povezati znanja iz dve teme (podudarnost trouglova i uglovi trougla; zbir uglova u trouglu i periferijski ugao...), tri teme (trougao, četvorougao, Pitagorina teorema; visina trougla, podudarnost trouglova, mnogougao...), ali i četiri teme (osobine pravouglog trougla, kružnica upisana u trougao, obim trougla, površina trougla; simetrala ugla, kružnica upisana u trougao, uglovi trougla, podudarnost trouglova). Analizom zahteva u ovim zadacima uočavamo da se sadržaji koje učenici treba da poznaju kako bi rešili date zahteve obrađuju gotovo u punom obimu u redovnoj nastavi. Ipak, neophodnost povezivanja sadržaja iz više oblasti i kreiranje različitih strategija jesu ono što njihovo rešavanje čini zahtevnim. Pored sposobnosti za prepoznavanje strategije i poznavanja potrebnih sadržaja, neophodna je i određena uvežbanost učenika, odnosno iskustvo u rešavanju složenih zadataka, koje se do određene mere stiče vežbom. Do takvih iskustava dolaze kroz saradnju sa vršnjacima koji imaju slična iskustva, ali i uz podršku nastavnika koji vodi učenike kroz postupak rešavanja, bar u prvom susretu sa takvim zadacima.

Na zadacima iz oblasti kombinatorike učenici SM odeljenja postigli su statistički značajno bolje rezultate u četiri zadatka, a skoro svi zadaci pripadali su oblastima dodatne nastave: kreiranje pobedničke strategije matematičke igre, primena kombinatorike za rešavanje problema bojenja, prebrojavanje palindroma, prebrojavanje načina na koji se elementi skupa mogu razbiti u podskupove (koji nisu obavezno disjunktni) sa zadatim svojstvom itd. Ovde se posebno ističe primena Dirihleovog principa u geometriji, kod kojeg je potrebno poznavanje geometrijskih sadržaja koji se obrađuju na redovnoj nastavi, ali je u rešavanju akcenat na prebrojavanju geometrijskih objekata i dokazivanju da je odabirom bilo kog određenog broja tih objekata moguće da se izabere njihov podskup takav da ispunjava zadati uslov.

Najmanje razlike u postignućima učenika su iz oblasti algebre i u ovoj oblasti se javljaju u 5 od 14 zadataka. U zadacima iz oblasti algebre akcenat je na primeni algebarskih transformacija, dokazivanju algebarskih jednakosti, određivanju vrednosti algebarskih izraza, primeni stepena i faktorizaciji prirodnog broja, što se obrađuje na redovnoj nastavi, dok se iz tema dodatne nastave izdvajaju rešavanje linearnih i nelinearnih Diofantovih jednačina, algebarski razlomci i složeniji zadaci iz stepena i operacije sa stepenom. Učenici SM odeljenja su statistički značajno bolje rešili zadatke u kojima je trebalo primeniti osobine stepena i algebarske transformacije, odrediti vrednost izraza u kojima se javljaju algebarski razlomci, faktorizaciju polinoma i određivanje skupa rešenja sistema nelinearnih jednačina. Možemo uočiti da se sadržaji zastupljeni u ovim zadacima pretežno javljaju na redovnoj nastavi, ali i da su zahtevi u njima prilično ozbiljni za taj uzrast učenika, budući da i podrazumevaju iskustvo u rešavanju sličnih zadataka, odnosno primenu odgovarajuće strategije rešavanja.

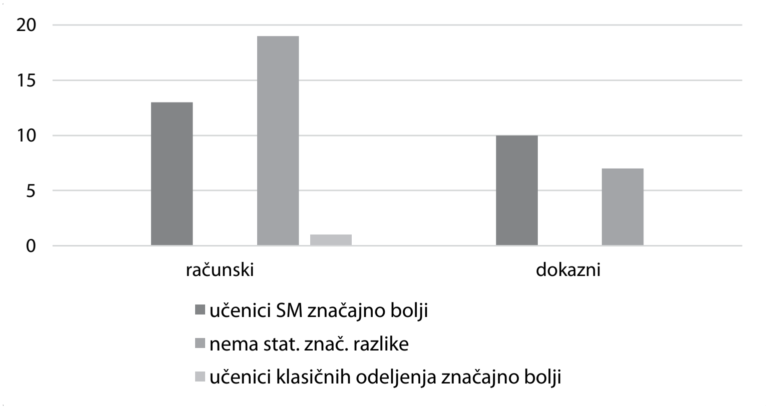
Uspeh učenika u zavisnosti od tipa zadataka

Pored matematičke oblasti kojoj pripadaju zadaci, želeli smo da analiziramo uspeh učenika SM i klasičnih odeljenja u odnosu na vrstu (tip) zadataka. U skladu sa time, podelili smo zadatke na računske i dokazne. Prvom tipu zadataka pripadaju svi zadaci u kojima je trebalo odrediti: vrednost brojevnog izraza, meru nekog geometrijskog objekta (dužinu duži, površinu i/ili obim figure, meru ugla), ostatak pri deljenju, nepoznatu vrednost i sl. Drugom tipu zadataka pripadaju oni zadaci u kojima se traži izvođenje neke vrste dokaza: jednakost mera geometrijskih objekata, deljivost algebarskog izraza prirodnim brojem, valjanost algebarskih jednakosti, egzistencija podskupova čiji elementi zadovoljavaju određeno svojstvo itd. Dokazni zadaci su obično zahtevniji od računskih, što potvrđuje i način bodovanja na određenim takmičenjima, gde se dokazni zadaci vrednuju značajno većim brojem poena (Facciaroni et al., 2023). U ovim zadacima učenici uglavnom prilaze rešenju tako što prvo opišu tvrđenje koje treba dokazati, pa zatim postupno izvode dokaz (Falk de Losada & Taylor, 2022). Postignuća učenika na osnovu ovako definisane klasifikacije data su na Slici 3.

Od 33 zadatka računskog tipa, učenici SM odeljenja ostvarili su statistički značajno bolja postignuća u 39,39% zadataka, u 57,58% zadataka nije bilo statistički značajne razlike, a samo jedan zadatak (3,03%) učenici iz klasičnih odeljenja su statistički značajno bolje uradili. Od preostalih 17 zadataka dokaznog tipa, učenici SM odeljenja su ostvarili statistički značajno bolje rezultate u 10 (58,82%) zadataka, dok u ostalim zadacima razlike nisu bile statistički značajne.

Grafikon 3

Postignuća učenika sedmog razreda na državnim takmičenjima u zavisnosti od tipa zadataka



Učenike koji pohađaju SM odeljenja, sa jedne strane, karakterišu dobre predispozicije za bavljenje matematikom na visokom nivou jer su neretko ostvarivali dobre rezultate na takmičenjima u šestom razredu, gde se već javljaju dokazni zadaci, a polaganjem kvalifikacionog ispita za upis u SM odeljenje potvrdili su svoje sposobnosti za rešavanje zahtevnih matematičkih zadataka. S druge strane, u okviru redovne i dodatne nastave, učenici SM odeljenja su zahvaljujući podršci nastavnika imali priliku da rešavaju veći broj dokaznih zadataka (u odnosu na učenike iz klasičnih odeljenja), čime su uvežbali sposobnost argumentacije, pa samim tim i naučili da, navođenjem odgovarajućih kontraprimera, pretežno dokazuju da zadatak nema rešenja ili da ne postoji matematički objekat koji zadovoljava zahtevana svojstva – osim što su svakako na časovima algebre i geometrije u većoj meri dokazivali poznata matematička tvrđenja.

Ukoliko posmatramo konkretne sadržaje kod dokaznih zadataka koje su učenici SM odeljenja bolje uradili, izdvaja se primena Dirihleovog principa u geometriji. Od tri zadatka primene Dirihleovog principa (jedan u teoriji brojeva, a dva u geometriji), razlike su u sva tri zadatka bile statistički značajne u korist učenika SM odeljenja (5. zadatak, 2018; 5. zadatak, 2020; 4. zadatak, 2022. godine). Poteškoća za učenike klasičnih odeljenja bila je generalizacija određene karakteristike matematičkih objekata. Upravo sposobnost generalizacije, kao misaoni proces, i Dirihleov princip, kao konkretan matematički sadržaj, predstavljaju kombinaciju gde učenici koji poseduju posebne sposobnosti za matematiku po pravilu ostvaruju bolja postignuća (Sriraman, 2003).

Zaključak

Rezultati ranijih istraživanja pokazuju da učenici sedmog razreda grupisani u homogena odeljenja, gde se u radu sa njima podrazumeva produbljivanje i obogaćivanje nastavnog programa, postižu statistički značajno bolje rezultate od svojih vršnjaka iz heterogenih odeljenja (Neihart, 2007; Swiatek & Lupkowski-Shoplik, 2003; Vulović i sar., 2024) na državnim takmičenjima. Do sličnih rezultata došli smo i našom analizom. Rezultati ukazuju da su učenici SM odeljenja postigli statistički značajno bolje rezultate u 46% zadataka na državnim takmičenjima u poslednjih deset godina. Statistički značajna razlika u korist učenika SM odeljenja se najviše puta javila u zadacima iz oblasti teorije brojeva, nakon čega slede geometrija i kombinatorika, da bi taj procenat zadataka bio najmanji iz algebre.

Za rešavanje zadataka iz određenih oblasti, poput teorije brojeva i kombinatorike, kao preduslov bilo je neophodno poznavanje određenih postupaka, tvrđenja i osobina matematičkih koncepata sa kojima se učenici u najvećoj meri sreću van redovne nastave matematike. Za njihovo usvajanje nije dovoljna sama izloženost tim sadržajima, već je neophodna i sposobnost argumentacije, uopštavanja i generalizacije. S druge strane, pored sposobnosti za prepoznavanje strategije i poznavanja sadržaja iz drugih oblasti, poput geometrije i algebre, potrebna je uvežbanost, odnosno iskustvo, za šta je uloga nastavnika koji treba da upozna učenike sa određenim procedurama u početku od krucijalnog značaja.

Kada je reč o tipu zadataka, u računskim zadacima su učenici SM odeljenja ostvarili statistički značajno bolje rezultate u nešto manje od 40% zadataka, dok je kod dokaznih zadataka taj procenat skoro 60%. Rešavanje dokaznih zadataka zahtevalo je temeljno poznavanje sadržaja iz više nastavnih tema, kao i sposobnost održavanja i sintetisanja toka misli u koherentan i logički zasnovan dokaz, koji je potkrepljen odgovarajućom argumentacijom. Dakle, potvrđeno je da poznavanje školske matematike nije dovoljno za uspešno rešavanje takmičarskih zadataka koji se zasnivaju na izvođenju dokaza i odgovarajućem rezonovanju prilikom tog postupka (Sal'kov et al., 2017).

Možemo zaključiti da su kombinacija prisustva nadarenosti učenika za matematiku, zajedno sa detaljnijim (obogaćenim i produbljenim) nastavnim programom, uz formalnije bavljenje određenim sadržajima doveli do toga da učenici SM odeljenja u znatnoj meri nadmašuju postignuća svojih vršnjaka iz klasičnih odeljenja. Pravac narednih istraživanja može se ogledati u sprovođenju kvalitativnih istraživanja kako bi se rasvetlilo koji je od ova dva efektadominantniji kod većeg broja učenika i u kojoj meri.

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Analysis of Seventh-Grade Students' Performance in National Mathematics Competitions by Topic and Task Type: A Comparison of Specialized and Regular Classes[1]

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Abstract *Mathematics competitions are increasingly attracting the researchers' attention in the field of mathematics education, as they offer numerous benefits to students — from deepening and broadening their knowledge to developing problem-solving skills. Given that seventh-grade students in the Republic of Serbia have had, the opportunity to attend specialized classes for mathematically gifted students (SM classes) for many years, the question arises as to whether grouping these students in homogeneous classes affects their performance in national mathematics competitions. The aim of this paper is to make a detailed comparison of the performances of students from SM and from regular classes in the national competitions and to analyze the tasks which the two groups solved with statistically significant differences. The analysis of students' performance in 50 competition tasks showed that SM students achieved statistically significantly better results in 23 tasks, while the students from the regular classes only performed better in one task. Greatest differences in favor of SM students were observed in number theory, followed by geometry and combinatorics, with the smallest differences in algebra tasks. SM students were significantly more successful in solving proof-based tasks than their peers from regular classes. A detailed analysis of tasks content suggests that the performance differences may stem from the broader and deeper mathematical education of SM classes, as well as their superior ability to synthesize various mathematical concepts and statements in the problem-solving process. The results of this study can serve as a guide for both teachers and students in preparing for mathematics competitions.*

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Introduction

Mathematics competitions for primary school students have a long-standing tradition in the Republic of Serbia. Since the first competitions in the 1960s and the first competition at national level in 1967 (Mićić i Kadelburg, 2022), they have been the most popular type of school competitions in the country. Each year, between 60,000 and 70,000 students from grades 4 to 8 take part in the first phase of the competition cycle (Vulović i sar., 2023).

The national mathematics competition is the fourth and final stage in which students in grades 6 to 8 participate. Although few studies have been conducted specifically on student performance in these competitions, the available research suggests that students who are taught in specialized mathematics (SM) classes for mathematically gifted students achieve statistically significantly better results than those in regular classes achieve. Vulović et al. (Vulović i sar, 2024) have empirically confirmed that the students from SM classes have performed better than their peers in regular classes, with statistically significant differences in 11 of the last 18 years.

This paper aims to investigate the performance of 7th grade students in national mathematics competitions from the years 2015 to 2024, focusing on mathematical domains and task types, depending on whether the students attend regular or SM classes. The study is based on the observation that the tasks in the curricula of the competitions are increasingly detached from the regular curriculum as they move from school to the international level of mathematics competitions. By analyzing the tasks that were predominantly solved by the highest-achieveing students in the observed period, we aim to contribute to the identification of key factors that influence students' performance in national mathematics competitions.

Theoretical background

One of the functions of mathematics competitions is to spark students' interest in engaging more deeply with mathematics and to foster a sense of competitiveness among them. Competitions provide support for students, as solving challenging tasks allows them to demonstrate their high mathematical abilities i.e., the abilities that are developed through the preparation process for such events (Keldibekova, 2020). Kenderov (2022) notes that solving complex mathematical problems not only leads to deeper mathematical understanding but also enhances students' skills in overcoming real-world problem situations.

Liljedahl (2019) distinguishes between "thinking classrooms" and "non-thinking classrooms"— that is, the learning environments which encourage students to learn through active thinking and those learning environments in which students merely imitate thinking. Mathematics competitions, in this sense, create an environment where student thinking is affirmed as a valuable experience for engaging with mathematics at a higher level. When discussing the competencies required for engaging with advanced mathematics, Falk de Losada & Taylor (2022) emphasize that solving competition problems helps develop students' problem-solving abilities and improves their understanding and use of mathematical terminology. While school mathematics typically follows a traditional structure of content delivery, mathematics competitions integrate content with modes of thinking and strategies for solving new and challenging problems. In doing so, they broaden students' perspectives and expose them to the reasoning methods used by professional mathematicians (Nieto-Said & Sanchez-Lamoneda, 2022).

The tasks in mathematics competitions typically fall within the domains of algebra, geometry, number theory, and combinatorics, although they often involve two or more domains simultaneously. The key to solving such tasks lies in ingenuity, creativity, and problem-solving skills. Research shows that the competition participants achieve varying levels of success depending on the predominant domain of the task. Students are often more successful in solving problems from algebra, combinatorics, and number theory compared to those from geometry (Facciaroni et al., 2023; Keldibekova, 2020; Špijunović i Maričić, 2013; Vulović i sar., 2024).

Facciaroni et al. (2023) examined differences in student performance across task domains using a sample of 29,592 ninth-grade students who participated in the regional level of mathematics competitions in Italy between 2018 and 2020. The study focused particularly on geometry and the difficulties which students encounter when solving geometric tasks and constructing formal proofs. Out of the seventeen tasks given in the competition each year, the first fourteen tasks did not require formal proof construction, although they did necessitate knowledge of theorems and procedures. The final three problems — one each from number theory, combinatorics, and geometry — did require formal proofs. These three tasks accounted for 40% of the total score, meaning that correctly solving them was decisive for qualifying for the national stage. The study found performance differences across mathematical domains, with a significant proportion of students not even attempting the proof-based geometry problem. In all three years covered by the study, the average score on the geometry proof task was significantly lower than the average scores on the other two proof tasks. In the same study, the authors also conducted a survey in 2020 involving 486 participants from the regional competition in the Rome district. On average, the students reported feeling equally comfortable when solving problems from each mathematical domain. Although not statistically significant, the differences in mean values indicated that students still perceived geometry as the most difficult domain, while number theory was seen as the easiest. The authors explain the small differences in students' attitudes by pointing out that nearly one-third of the students responded that the competition problems in geometry were similar in complexity, content, or structure to those which they had encountered during regular classes. In contrast, only about one in five students on average gave the same response for the other three domains. This supports the finding that, even though students — especially the high-achieving ones — are familiar with the basic geometric rules taught in regular classes, they still encounter difficulties when solving competition-level geometry problems (Sal'kov et al., 2017).

Although solvable using elementary methods, problems in mathematics competitions can be very challenging. Unlike conventional school mathematics problems, which typically require reproducing known procedures, competition problems are inclusive, since they consider various situations, conjectures, examination of assumptions, and posing of new questions (Leikin, 2007). A solution is considered complete if and only if the student provides a full proof of their claim. Falk de Losada & Taylor (2022) note that students in competitions, similarly to professional mathematicians, most often begin the proving process by stating what is to be proven (proposing an argument), and then proceed with the construction of the proof. In doing so, students employ one of four types of proofs (Balacheff, 1988, as cited in Laamena et al., 2018): naive empiricism, where the truth of the argument is confirmed after verifying special cases; key experiment, which is used to choose between two previously formulated hypotheses; generic example, which explains why the argument holds true by citing specific properties and structures; thought experiment, which generalizes the argument and distinguishes it from particular cases.

The impact of student grouping on academic achievement has been widely discussed in the literature; however, the influence of homogeneous grouping on student performance in mathematics competitions has been less extensively studied. The primary goal of this type of grouping is to meet the interests and specific educational needs of mathematically gifted students, who most frequently participate in mathematics competitions. Karp (2009) states that instruction in schools where mathematically gifted students are grouped together is characterized by problem posing, problem solving, seeking original methods of solving problems, and finding connections between new and old problems. Kulik & Kulik (1987) conducted a meta-analysis including 90 studies, 25 out of whom focused specifically on examining the effects of instruction in classes composed of gifted students. In 19 of these studies, results showed that gifted students achieved better outcomes when grouped homogeneously, and in 11 of these studies, the differences were statistically significant. The same authors also note that grouping gifted students in homogeneous classes does not yield the desired effects if an appropriate curriculum transformation to meet their educational needs is lacking. These findings are supported by the results of a study by Steenbergen-Hu et al. (2016), which covered one hundred years of research on the effects of student grouping on academic achievement.

A positive impact of transferring students from regular classes to gymnasiums that offer instruction for primary school students gifted in mathematics was also observed in the study by Asmat (2021). Since students can enroll in such classes after completing either the fifth or eighth grade, one of Asmat's research objectives was to determine the extent to which school choice and class type influence student achievement in competitions. It was found that students attending gymnasiums achieve on average 6% higher results than those from heterogeneous classes. This finding confirms the advantages of the instructional approach applied in homogeneous classes, where the focus is placed on an enriched mathematics curriculum (Kim, 2016; Rochayati & Rochayani, 2024).

Sriraman (2003) notes that complex mathematical problems provide an excellent opportunity for students to develop higher-order mathematical processes, primarily representation, abstraction, and generalization. In his study, Sriraman assigned the task of solving five non-routine combinatorial problems to a group of nine ninth-grade students, in such manner that the generalization required for solving these problems was presented in the form of the Pigeonhole principle. Four students, previously identified as mathematically gifted, successfully discovered and formulated the generalization needed to solve the five problems, while the other five students, who did not possess special mathematical abilities, failed to recognize the generalization hidden in the problems (Sriraman, 2003).

The essence of grouping mathematically gifted students is not to isolate them from average-ability peers, but to unite them with those who share similar interests. When grouped together, students are stimulated by peers of comparable ability and can process any more complex content more quickly and effectively (Neihart, 2007; Swiatek & Lupkowski-Shoplik, 2003). Mathematics instruction in such classes is characterized by strategies of acceleration, enrichment, and deepening.

In their research, Vulović et al. (2024) analyzed the partcipation frequency and achievements at national mathematics competitions in Serbia over an eighteen-year period of seventh- and eighth-grade students attending specialized mathematics (SM) classes in gymnasiums and of students from heterogeneous classes. The authors emphasize that in both seventh and eighth grades, the average scores achieved by students in SM classes were higher each year compared to those from regular classes. On average, seventh-grade students in SM classes scored 39% higher than their counterparts in regular classes, while the achievement of eight-grade students was 32% higher.

Research Methodology

The law in the Republic of Serbia recognizes the possibility of grouping mathematically gifted students into special classes. After completing the sixth grade of primary school, students have the option to transfer to gymnasiums that offer instruction tailored for students gifted in mathematics (Guidelines on the Curriculum for Seventh and Eighth Grade Students of Primary Education Gifted in Mathematics, 2019). Upon enrollment, students take a special qualifying exam that evaluates their performance on a mathematics ability test, overall grades from fifth and sixth grades, mathematics grades in those years, and participation in the national mathematics competition in the sixth grade.

The national-level mathematics competition takes place every year in May, and approximately one hundred students from each of the sixth, seventh, and eighth grades of primary school, who achieve the highest results at the preceding district-level competitions during that school year, are eligible to participate.

The aim of this research is to analyze the performance of seventh-grade students from specialized mathematics (SM) and regular classes in national mathematics competitions with respect to the mathematical domains to which the competition tasks belong as well as to the types of given tasks. In line with this aim, differences in the performance of students from regular and SM classes in solving these tasks were examined, along with an analysis of the content of the given tasks. The research objectives derived from this aim are to determine:

1. The ratio of the number of tasks in which SM students achieved better results than students from regular classes, the number of tasks in which there were no differences in performance between the two observed groups, and the number of tasks in which students from regular classes achieved better results than SM students;
2. The ratio of the number of tasks in the context of the performance of SM and regular class students depending on the mathematical domain to which the tasks belong;
3. The ratio of the number of tasks in the context of the performance of SM and regular class students depending on whether the tasks are computational or proof-based.

The paper presents a secondary analysis of the performance of seventh-grade students in national mathematics competitions from the year 2015 to 2024. At the competitions, students solved five tasks prepared by the State Commission for Primary School Competitions, with a time limit of 180 minutes for completion. All tasks are publicly available on the official website of the Serbian Mathematical Society. The national competition is held live each year, as are all preceding levels, in school settings. The content of the tasks is defined by the Competition Program issued by the Serbian Mathematical Society at the beginning of the competition cycle, based on the Competition Rulebook. Both the Rulebook and the Competition Program are publicly accessible. Task evaluation is conducted according to a precisely defined scoring scale and grading instructions (partial credit is awarded). Each task is scored with a maximum of 20 points.

The results obtained from the national competitions each year are consolidated into a unified database, which was used for this study. The research sample consists of all seventh-grade students who participated in the national mathematics competitions from 2015 to 2024, totalling 1,014 students. The population structure, with respect to the type of class attended by the students, is presented in Table 1.

Table 1
Structure of the research population

Year	Regular class		SM class	
	F	%	*F*	%
2015.	64	66,67%	32	33,33%
2016.	50	50,00%	50	50,00%
2017.	57	58,16%	41	41,84%
2018.	53	52,48%	48	47,52%
2019.	60	62,50%	36	37,50%
2020.	60	61,22%	38	38,78%
2021.	50	50,00%	50	50,00%
2022.	61	54,46%	51	45,54%
2023.	64	61,54%	40	38,46%
2024.	67	61,47%	42	38,53%
Total	586	57,79%	428	42,21%

The collected data were processed using the SPSS 20 software package. The statistical measures and procedures used included frequencies, percentages, arithmetic mean, the Shapiro-Wilk test for assessing the normality of numerical data distribution, the Student's t-test for independent samples, and the Mann-Whitney test for comparing data distributions between two groups.

Results and Discussion

In line with the research objective, we analyzed whether statistically significant differences exist in the performance of students attending specialized mathematics (SM) and regular classes in solving seventh-grade tasks at national mathematics competitions, examining each individual task over the past ten years (a total of 50 tasks). For each task, it was tested whether the scores that were achieved by students in both groups followed a normal distribution. In cases when both groups exhibited normal distribution, the arithmetic means of the scores were compared using the independent samples Student's t-test; otherwise, the distributions of scores achieved by students in the two groups were compared using the Mann-Whitney test. The results are presented in Table 2. For each task, the arithmetic means of the scores achieved by students from both groups are given, while the results of the Mann-Whitney test are reported in cases where statistically significant differences were found. No statistically significant differences in student performance were found in any task where the Student's t-test was applied.

Table 2
Performance of seventh-grade students by class type, per task, in the period from 2015 to 2024

Year	Task Number									
	1.		2.		3.		4.		5.	
	Regular	SM	Regular	SM	Regular	SM	Regular	SM	Regular	SM
2015.	5,72	8,97	**2,73**	**5,56**	2,39	4,09	6,42	9,13	1,31	3,50
			U=798.0 Z=-3.100 p=.002							
2016.	7,90	8,38	9,86	12,72	**3,44**	**8,78**	**12,00**	**15,98**	3,64	4,56
					U=914.0 Z=-2.414 p=.016		**U=1009.5 Z=-1.994 p=.046**			
2017.	**7,00**	**11,56**	**4,30**	**9,85**	**4,65**	**9,90**	**3,21**	**8,02**	**7,18**	**11,32**
	U=798.0 Z=-2.766 p=.006		**U=644.5 Z=-3.946 p=.000**		**U=782.5 Z=-2.833 p=.005**		**U=861.5 Z=-2.349 p=.019**		**U=849.0 Z=-2.371 p=.018**	
2018.	*17,08*	*16,96*	**8,58**	**12,54**	2,17	2,35	3,02	4,73	**6,57**	**13,42**
			U=990.0 Z=-2.071 p=.038						**U=742.0 Z=-3.800 p=.000**	
2019.	10,32	13,39	**6,75**	**12,17**	10,00	12,06	6,90	8,06	**7,03**	**3,72**
			U=664.5 Z=-3.178 p=.001						**U=862.0 Z=-1.962 p=.050**	
2020.	9,62	13,08	**8,55**	**13,95**	**10,50**	**18,71**	**2,67**	**6,58**	**4,40**	**9,76**
			U=745.0 Z=-3.056 p=.002		**U=591.0 Z=-4.545 p=.000**		**U=829.5 Z=-2.394 p=.017**		**U=690.0 Z=-3.491 p=.000**	
2021.	6,00	7,60	11,06	12,18	**11,10**	**15,62**	14,22	15,54	*2,00*	*1,20*
					U=865.0 Z=-2.987 p=.003					
2022.	**9,18**	**12,65**	**2,26**	**7,94**	4,25	4,90	**8,08**	**12,84**	**0,84**	**5,35**
	U=1193.5 Z=-2.306 p=.021		**U=866.5 Z=-4.523 p=.000**				**U=1063.5 Z=-2.960 p=.003**		**U=1146.5 Z=-3.147 p=.002**	
2023.	6,53	8,75	4,48	7,03	10,08	12,15	4,84	5,50	1,39	3,57
2024.	*18,33*	*17,83*	**10,21**	**15,14**	9,76	11,57	**2,96**	**7,07**	**11,30**	**14,38**
			U=931.0 Z=-3.064 p=.002				**U=1011.5 Z=-2.781 p=.005**		**U=1080.0 Z=-2.095 p=.036**	

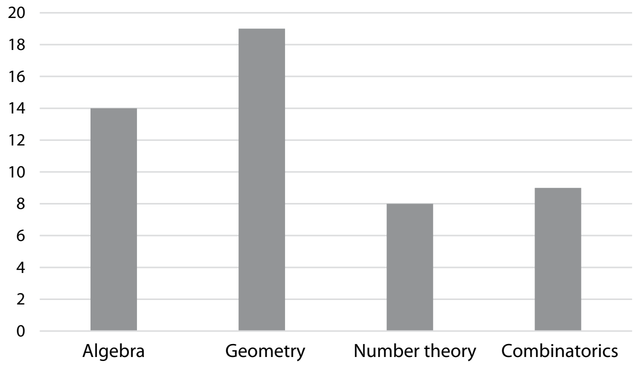
Out of 50 observed tasks, students from specialized mathematics (SM) classes achieved statistically significantly better results in 23 tasks, the differences in 26 tasks were not statistically significant, while students from regular classes performed statistically better in 1 task.

Student performance depending on the mathematical domains of the tasks

It has already been mentioned that the tasks belong to the domains of algebra, geometry, number theory, and combinatorics, and that solving certain tasks requires knowledge and skills from two or three of these domains. Each task was categorized into the domain that is dominant in its solution, based on content and procedures. According to Figure 1, during the observed period, the largest number of tasks at the national competitions belonged to geometry, followed by algebra, then combinatorics, with the fewest tasks coming from number theory.

Figure 1

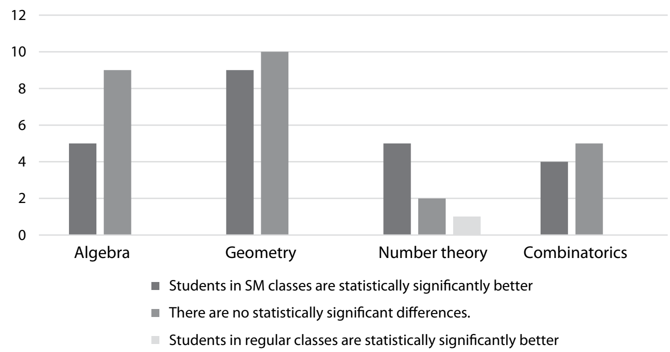
Number of tasks at national competitions for 7th-grade students from 2015 to 2024, by mathematical domain



If we compare the task performance of students from specialized mathematics (SM) and regular classes according to the mathematical domain to which the tasks belong, we obtain the results shown in Figure 2.

Figure 2

Performance of seventh-grade students from specialized mathematics (SM) and regular classes in national competitions from 2015 to 2024, by mathematical domain



We observe that the difference in performance favoring students from specialized mathematics (SM) classes is the greatest on the tasks from number theory. Out of 8 tasks in this domain, SM students achieved statistically significantly better results on five. Analyzing the content of these tasks, we notice that of them all include parts which can be solved using knowledge from regular instruction, such as prime factorization, divisibility, and remainders in division by natural numbers. However, a successful completion of the entire solution usually requires good knowledge of additional instruction content, for example, decoding numbers in decimal notation according to given conditions or applying Pigeonhole's (combinatorial) principle (to divisibility by a natural number). For instance, the use of congruences to determine the remainder in division and specifically to find the last digit of a multi-digit number (a special case of division by 10) is something the average primary school student does not encounter unless the teacher dedicates special attention to it during additional lessons. On the other hand, seventh-grade students in SM classes are more likely to have the opportunity to learn the properties of congruences, as these topics appear in their curriculum. Furthermore, Pigeonhole's principle is also covered exclusively in additional mathematics lessons (and emphasized in the competition program). It is worth noting that the tasks applying Pigeonhole's principle are classified under combinatorics, but one specific task (from 2018) was classified under number theory because the focus of the solution was on applying divisibility properties.

In geometry tasks, students from specialized mathematics (SM) classes achieved statistically significantly better results on 9 tasks, while no significant difference was found in ten tasks. The content represented in these tasks was mostly learned before seventh grade (properties of squares and rectangles, triangle angles, triangle congruence, areas of triangles and quadrilaterals, axial symmetry, segment bisector, angle bisector, inscribed and circumscribed circles in triangles, etc.). Content from the seventh-grade curriculum included the Pythagorean theorem with applications to triangles, quadrilaterals, and polygons, circles, inscribed and central angles. It is especially important to emphasize that all tasks required knowledge of multiple definitions, theorems, and properties from various topics, and the ability to synthesize all of it into a complete solution. Some tasks required connecting knowledge from two topics (e.g., triangle congruence and triangle angles; sum of angles in a triangle and inscribed angle), three topics (e.g., triangle, quadrilateral, Pythagorean theorem; triangle height, congruence, polygon), and even four topics (e.g., properties of right triangles, inscribed circle in a triangle, triangle perimeter, triangle area; angle bisector, inscribed circle, triangle angles, congruence). Analyzing the demands of these tasks, we note that the much of the required content which students need to know is largely covered in regular instruction. However, the challenge lies in the need to connect content from multiple areas and create different solving strategies. Besides the ability to recognize strategies and knowledge of necessary content, students also require a certain level of practice and experience in solving complex problems, which is gained through repeated exercises. This experience is further developed through collaboration with peers who share similar experiences, as well as with the support of teachers who guide students through the solving process, at least during their first encounters with such tasks.

In combinatorics tasks, students from specialized mathematics (SM) classes achieved statistically significantly better results on 4 tasks, with nearly all tasks belonging to topics covered in supplementary lessons: creating winning strategies for mathematical games, applying combinatorics to solve coloring problems, counting palindromes, counting the ways elements of a set can be partitioned into subsets (not necessarily disjoint) with given properties, and so on. It is particularly noteworthy is the application of the Pigeonhole Principle in geometry, which required knowledge of geometric content taught in regular classes, but in solving these tasks the focus was on counting geometric objects and proving that, for a chosen number of such objects, it is possible to select a subset that satisfies a given condition.

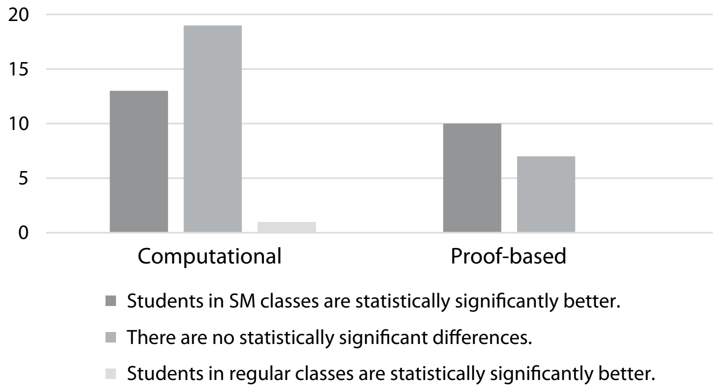
The smallest differences in student performance were observed in the domain of algebra, where differences occur in 5 out of 14 tasks. Algebra tasks were focused on applying algebraic transformations, proving algebraic equalities, determining the values of algebraic expressions, applying exponents and factoring natural numbers, all of which are covered in regular classes. Topics from supplementary lessons include solving linear and nonlinear Diophantine equations, algebraic fractions, and more complex problems involving exponents and exponent operations. Students from specialized mathematics (SM) classes performed statistically significantly better on the tasks requiring the application of exponent properties and algebraic transformations, determining values of expressions containing algebraic fractions, factoring polynomials, and finding solution sets of nonlinear equation systems. While the content in these tasks generally aligns with regular instruction, the level of difficulty is quite advanced for this age group and again suggests the importance of prior experience in solving similar problems, by applying appropriate solution strategies.

Student performance depending on task type

In addition to the mathematical domains to which the tasks belong, we aimed to analyze the performance of students from specialized mathematics (SM) and regular classes with respect to the type of task. Accordingly, we divided the tasks into computational and proof tasks. The first type includes all tasks where students had to determine: the value of a numerical expression, the measure of a geometric object (length of a segment, area and/or perimeter of a figure, measure of an angle), remainder upon division, unknown value, etc. The second type includes tasks requiring the construction of some form of a proof: equality of measures of geometric objects, divisibility of an algebraic expression by a natural number, validity of algebraic equalities, existence of subsets whose elements satisfy certain properties, and so on. Proof tasks are generally more demanding than computational ones, which is also confirmed by the scoring system in certain competitions where proof tasks are awarded significantly more points (Facciaroni et al., 2023). In these tasks, students typically approach the solution by first stating the claim to be proven, and then gradually constructing the proof (Falk de Losada & Taylor, 2022). The students' performances based on this classification are shown in Figure 3.

Of the 33 computational-type tasks, students from the specialized mathematics (SM) classes achieved statistically significantly better results in 39.39% of the tasks, there was no statistically significant difference in 57.58% of the tasks, while only one task (3.03%) was completed statistically significantly better by students from regular classes. Among the remaining 17 proof-type tasks, students from SM classes achieved statistically significantly better results in 10 tasks (58.82%), while in the remaining tasks the differences were not statistically significant.

Figure 3
Performance of seventh-grade students from specialized mathematics (SM) and regular classes in national competitions from 2015 to 2024, by task type



Students attending specialized mathematics (SM) classes, on one hand, are characterized by strong predispositions for engaging with mathematics at a high level, as they often achieved good results in competitions in the sixth grade, where proof-based tasks already appear. By passing the qualifying exam for admission to SM classes, they demonstrated their ability to solve demanding mathematical problems. On the other hand, through regular and supplementary instruction, SM students — supported by their teachers —had more opportunity to solve a greater number of proof-based tasks (compared to students from regular classes), thereby strengthening their argumentative skills. They were also more extensively exposed to learning how to use appropriate counterexamples in order to prove that a task has no solution or that no mathematical object satisfies the required properties. Additionally, during algebra and geometry lessons, they proved known mathematical theorems to a greater extent.

If we consider the specific content of the proof-based tasks on which SM students outperformed their peers, the application of the Pigeonhole principle in geometry stands out. Among the three tasks involving the Pigeonhole principle (one in number theory and two in geometry), the differences were statistically significant in all three tasks in favor of the SM students (task 5 from 2018; task 5 from 2020; task 4 from 2022). The difficulty for students from regular classes was the generalization of mathematical objects' certain characteristics. The ability to generalize, as a cognitive process, combined with the Pigeonhole principle as a specific mathematical concept, represents a challenge in which mathematically gifted students typically perform better (Sriraman, 2003).

Conclusion

Results from previous studies show that seventh-grade students grouped in homogeneous classes, where intensification and enrichment of the curriculum is assumed in their instruction, achieve statistically significantly better results in national competitions than their peers from heterogeneous classes (Neihart, 2007; Swiatek & Lupkowski-Shoplik, 2003; Vulović et al., 2024). Our analysis yielded similar results. The findings indicate that students in SM classes achieved statistically significantly better results in 46% of the tasks at national competitions over the past ten years. The greatest number of statistically significant differences in favor of SM students occurred in tasks from the field of number theory, followed by geometry and combinatorics, with the lowest percentage in algebra.

Solving problems in certain domains, such as number theory and combinatorics, requires prior knowledge of specific procedures, theorems and properties of mathematical concepts, many of which students mostly encounter outside of regular mathematics classes. However, mere exposure to this content is not sufficient for mastery; rather, students also need the ability to argue, to generalize and to abstract. On the other hand, besides the ability to recognize strategies and knowledge of content from other domains, like geometry and algebra, practice and experience are required. At the initial stages, the teacher plays a key role, by introducing students to the relevant procedures and guiding them through the problem-solving process. Regarding the type of the tasks, students in SM classes achieved statistically significantly better results in just under 40% of computational tasks, while for proof-based tasks that percentage was nearly 60%. Solving proof-based tasks required a thorough knowledge of content from multiple instructional topics, as well as the ability to maintain and synthesize the flow of thought into a coherent and logically sound proof which is supported by appropriate argumentation. Thus, it was confirmed that knowledge of school mathematics alone is not sufficient for a successful solution of competition tasks that are based on constructing proofs and appropriate reasoning during the process (Sal'kov et al., 2017).

We can conclude that the combination of students' mathematical giftedness, together with a more detailed (enriched and intensified) curriculum and a more formal engagement with specific content, has led to students in SM classes significantly outperforming their peers in regular classes. Future research could take the form of qualitative studies aimed at clarifying which one of these two effects is more dominant for a larger number of students and to what extent.

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