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COMPLEX STRESS-STRAIN BEHAVIOUR OF A CYLINDRICAL SHELL WITH A DYNAMICALLY BREAKING INTERNAL ELASTIC BASE

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During the operation of a solid-propellant rocket engine, the combustion products of a powder charge create increased pressure in the combustion chamber. Besides, the combustion of gunpowder is accompanied by a large release of heat, which, despite the thermal insulation, causes the appearance of deformations in the engine cowling. This leads to the need to investigate the durability of the shell under the influence of internal pressure and temperature fields. The aim of the paper is to determine the complex dynamic deformed state and vibrations of the engine cowling under the action of force and temperature loads. The problem of a complex axisymmetric stress-strain state and vibrations of a thin cylindrical shell with a dynamically breaking internal elastic foundation, obeying Winkler's hypothesis, is approximately solved. The shell is under the action of internal pressure and temperature fields on a part of its length free from an elastic base. The resolving equation of the problem of the shell deflection is solved by the Bubnov-Galerkin method, reducing the problem to a system of linear algebraic equations. The examples are considered, in which the basic frequencies of natural vibrations of the structure are determined depending on the conditions of shell fastening. Parametric studies are carried out.

Key words: partial internal elastic foundation, combined loading, axisymmetric deformation, elevated temperature, vibration frequencies

INTRODUCTION

The problem of dynamic deformation of a cylindrical shell of a solid-propellant engine under the influence of internal pressure of gases (combustion products) and temperature fields that change in time as the powder charge burns out is of immense complexity. Its solution can be obtained only numerically [1-3]. However, at the stage of preliminary research, when the design parameters are not yet precisely known, calculations by numerical methods such as, for example, the finite element method is impossible [4, 5]. Therefore, it is necessary to obtain preliminary results of its strength static and dynamic tests allowing to qualitatively and quantitatively evaluate the influence of certain parameters on the studied characteristics to solve the problem in a simplified formulation.

The most important dynamic characteristics of the engine cowling are the natural frequencies of its oscillations, and especially its lowest (fundamental) frequency [6]. If it or some other natural frequencies coincide with the frequencies of forced vibrations of the engine in its operation mode, arising as a result of uneven (pulsating) combustion of the powder charge, then this inevitably leads to the occurrence of resonant vibrations [7-9]. And this is a direct and short path to the destruction of the entire structure. Therefore, the proposed approximate solution to the problem under consideration is of great practical importance and is relevant for the design of solid fuel engines.

The natural vibrations of the engine cowling are influenced by its various design parameters and the characteristics of the powder charge (stiffness coefficients). Problems of this kind were considered in [10-12]. In addition, the idealisation of the choice of boundary conditions at the end walls of the cylinder has the influence.

Therefore, the purpose of the study is to determine the complex stress-strain behaviour and vibrations of the engine cowling under the action of a combination of force and temperature loads. The main objectives are: simulate the engine cowling with a thin cylindrical shell; to study the dependence of the fundamental frequency on the degree of burnout of the powder charge; numerically investigate the dependence of the fundamental frequency on the stiffness coefficient of the powder charge.

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MATERIALS AND METHODS

The proposed solution to the problem is based on the use of the theory of a thin-walled elastic circular cylindrical shell in an axisymmetric stress-strain state and obeying Kirchhoff-Love's hypotheses [13]. During the operation of the engine, the fuel partially burnt out, and the area where it is stored continuously decreases. The latter circumstance leads to the need to consider the solution of the problem in time. The adhesion of the powder charge and the inner walls of the engine cowling is modelled on the basis of Winkler's hypothesis, considering the fuel as an elastic foundation. In accordance with this hypothesis, the loads on the cladding are proportional to its deflections and the stiffness coefficient of the fuel [14, 15]. The boundary of the fuel burnup zone at any moment of time is determined through the generalised Heaviside function. In addition, the internal pressure of the propellant gases (combustion products) and the increased temperature resulting from the burnout of the charged act on the freed part of the engine housing. All this taken together causes a complex deformation of its walls. The area of action of these loads is also determined through the Heaviside functions.

Thus, the problem is reduced to solving the differential equation of motion of the shell in partial derivatives under the action of loads of various physical nature. Unknown in this equation is shell deflection. It has the fourth order in the longitudinal coordinate and the second in time. In addition, it contains breaking factors associated with the description of loads using generalised functions. Its exact solution is generally impossible. Therefore, for an approximate solution, we use the Bubnov-Galerkin method. In accordance with it, we represent the shell buckling in the form of a series of specified functions with unknown coefficients that vary harmonically in time. Since this method is only an approximate method for solving differential equations, the used approximating functions must satisfy both geometric and force boundary conditions at the edges of the shell. As a result, the problem is reduced to a system of linear algebraic equations for the unknown expansion coefficients of the shell deflection. The presence of discontinuous coefficients in the original equation leads to the fact that, even by choosing the approximating functions as orthogonal, the resulting system of algebraic equations does not split into separate independent equations [16-18].

When solving the problem of static strength, the equations will be inhomogeneous and their solution is carried out by methods of linear algebra. When solving the problem of dynamics, the system becomes homogeneous and contains the oscillation frequency as a parameter. They are determined from the condition that the determinant of this system is equal to zero. In high approximations, they can only be found numerically. However, the most important from the standpoint of practice, the lowest (fundamental) frequency can be found in a monomial approximation in a closed-form, keeping only one term of the series in the solution. It essentially depends, among other things, on the adopted boundary conditions at the edges of the shell. This dependence is demonstrated in the paper on specific numerical examples.

RESULTS AND DISCUSSION

To solve the problem, we simulate the engine cowling with a thin cylindrical shell, inside on a part of its length there is an elastic base that burns out with a constant velocity *V* over the cross-sectional area corresponding to the powder charge (Fig. 1). The internal pressure of intensity p and the temperature field *F(x)* varying along with the generatrix act on the part of the shell free from it. As a result of heat exchange, the section of the shell, where the fuel is still preserved, also heats up, but according to a different law *f(x-Vt)*. Equality remains at the boundary of the combustion zone *F(Vt)=f(0)*. The laws of temperature change in these areas can be found from the equations of heat conduction, but this issue is not studied in our case. The temperature is evenly distributed over the thickness of the cladding, and the time for complete fuel burnout is $t_0=L/V$.

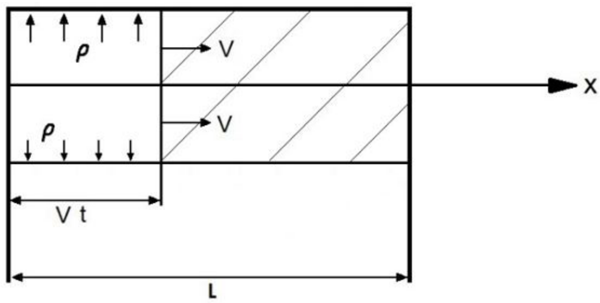

Figure 1: Shell with partially burnt filler

Figure 1 shows the state of the structure at an arbitrary moment of time t $(0<t<t_0)$. The part of the elastic base that has survived to this moment is cross-hatched. Under the action of internal pressure and temperature fields, the shell undergoes an axisymmetric deformation that changes only along the *x* axis. The region of internal pressure on the shell and the zone of elevated temperature is set the difference between the Heaviside functions *H(x-0)* and *H(x-Vt)* given at the points *x=0* and *x=Vt* respectively.

The elastic base is considered to be inertial, and the contact between it and the cylinder wall is continuous. Therefore, the pressure on the shell in this zone is determined by Winkler's hypothesis, considering it to depend only on the shell deflection *w* and the base stiffness coefficient *b*. Then the forces of interaction between the shell and the powder charge are equal to *wb*, and the boundary of their range is determined using the Heaviside function *H(x-Vt)* [19, 20]. Taking into account all of the above, we investigate the axisymmetric deformation of the cylinder on the basis of the moment theory of shells, the resolving equation of which for the case under consideration takes the form (Eq. 1):

$$\frac{\partial^4 w}{\partial x^4}+\frac{m}{D}\frac{\partial^2 w}{\partial t^2}+w(4\beta^4+\frac{b}{D}H(x-Vt))=P_1(x,t)-P_2(x,t) \quad (1)$$

where (Eqs. 2-3):

$$P_1(x,t)=\frac{1}{D}(H(x-0)-H(x-Vt))\times(p-\frac{Eh\alpha}{R}F(x)), \quad (2)$$

$$P_2(x,t)=\frac{Eh\alpha}{DR}f(x-Vt)H(x-Vt) \quad (3)$$

Where P_1 *(x,t)* – power and temperature loads acting on the part of the cylinder free from the base *(x<Vt)*, P_2*(x,t)* – purely thermal load acting in the elastic foundation zone. In expressions (1) and (2-3) it is noted: $D=Eh^3/12(1-\mu^2)$ cylindrical stiffness of the shell, *m=ρh* – its linear mass, $\beta^4=3(1-\mu^2)/R^2 h^2$, *R, h* – shell radius and thickness, *E, μ, ρ* and *α* – modulus of elasticity, Poisson's ratio, density and coefficient of linear expansion of its material, respectively.

It is possible to build a solution without using generic Heaviside functions [21]. In this case, instead of one equation (1), it is necessary to use two equations, one of which describes the deformation of the cylinder at *x<Vt* and *b=0*. In this case, the load is determined by the function P_1 *(x,t)*. The second equation finds the deformation in the contact zone with the elastic base at *Vt<x<L*, and the load, in this case, is determined by the function P_2 *(x,t)*. With this solution, the deflections in two sections must be joined in the section of the cylinder with the coordinate *x=Vt*.

However, using one equation instead of two is preferable from a purely computational standpoint. Therefore, we construct a further solution to the problem based on equation (1). It is a partial differential equation containing discontinuous coefficients with Heaviside functions that determine at each moment of time the boundary of the zone where the elastic foundation is located. We solve the equation by the Bubnov-Galerkin method, according to which we represent the shell deflection w in the form of the expansion (Eq. 4):

$$w(x,t)=\sin\omega t\sum_{i=1}^{N} w_i\phi_i(x) \quad (4)$$

where ω – vibration frequency, w_i – unknown coefficients, ϕ_i(x) – coordinate functions, satisfying the boundary conditions at the ends of the shell.

Applying the procedure of the Bubnov-Galerkin method to equation (1), we reduce the problem to a coupled system of linear algebraic equations with respect to w_i. In matrix representation, it has the form (Eq. 5):

$$|K-\omega^2 M|W=C \quad (5)$$

The matrices of mass *M*, stiffness *K*, and vectors *W* and *C* included in (4) have the form (Eqs. 6-8):

$$M=[m_{ij}] \quad (6)$$

$$K=[k_{ij}] \quad (7)$$

$$W=\{w_j\}, C=\{c_j\} \quad (8)$$

The dimension of matrices and vectors is determined by the number of terms of the series stored in the expansion (4), and their elements are equal (Eqs. 9-11):

$$k_{ij}=\left\{\int_0^L \phi_i^{IV}\phi_j dx+4\beta^4\int_0^L \phi_i\phi_j dx+\frac{b}{D}\int_0^L H(x-Vt)\phi_i\phi_j dx\right\} \quad (9)$$

$$m_{ij}=\int_0^l \phi_i\phi_j dx \quad (10)$$

$$c_j=\frac{1}{D}\int_0^L (H(x-0)-H(x-Vt)(p-\frac{Eh\alpha}{R}F(x))\phi_j dx - \frac{Eh\alpha}{DR}\int_0^L f(x-Vt)H(x-Vt)\phi_j dx \quad (11)$$

The dashes above the function ϕ_i denote its derivatives concerning the longitudinal coordinate x. Even when the approximating functions ϕ_i *(x)* in the expansion (4) are chosen to be orthogonal due to the presence of generalised Heaviside functions in the stiffness coefficients k_{ij}, the system of equations (5) does not split into separate unrelated equations. Therefore, when solving the problem in high approximations, equation (5) can be solved only numerically. After solving it following expansion (4), it is possible to reconstruct the deflection function of the shell *w*, and then, using the formulas of the shell theory, find the stresses. To determine the natural frequencies of vibrations of a structure at an arbitrary moment in time $0<t<t_0$ it is necessary to equate to zero the determinant of the system of equations (5) (Eq. 12):

$$\omega_1^2=\frac{k_{11}}{m_{11}}=\frac{\int_0^L \phi_1^{IV}\phi_1 dx+4\beta^4\int_0^L \phi_1^2 dx+\frac{b}{D}\int_0^L H(x-Vt)\phi_1^2 dx}{\frac{m}{D}\int_0^L \phi_1^2 dx} \quad (12)$$

In practice, the most important basic (lower) natural vibration frequencies correspond to the simplest forms of shell motion [22, 23]. Therefore, as an example, in a one-term approximation using formula (13), we will determine the basic vibration frequencies of the structure under consideration at various fastenings at the ends of the shell. The current time and rate of fuel burnup are included in the solution through a complex parameter *Vt/L*, showing the dimensionless length of the fuel section that has burned out by the considered moment. In the case of free support of both edges of the cylinder at *x=0* and *x=L*, we take the approximating function in the expansion (4) in the form (Eq. 14):

$$\phi=\sin(\pi x/L) \quad (13)$$

and with rigid fastening of its ends – in the (Eq. 15):

$$\phi=1-\cos(2\pi x/L), \quad (14)$$

Initial data for calculations are presented in the dimensionless form: *L/R=10, R/h=50*. Figures 2 and 3 show the dependence of the dimensionless square of the fundamental natural vibration frequency of the structure (Eq. 16):

$$\omega_*^2=\omega^2\rho R^2/E \quad (15)$$

on the parameter *Vt/L* and dimensionless coefficient of the stiffness of the elastic foundation respectively (Eq. 17):

$$b_*=bR/E \quad (16)$$

In the Figures 2-3, curves 1 correspond to the rigid fastening of the ends of the shell, and curves 2 – to their free

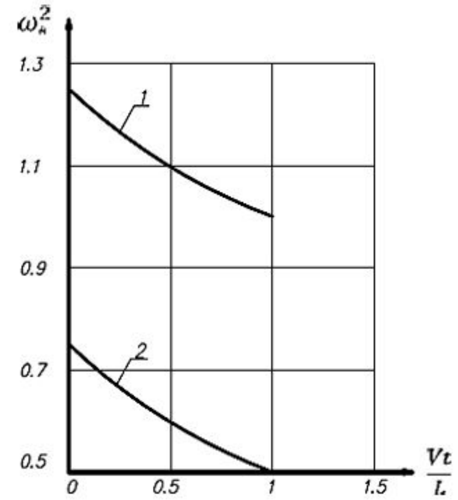

Figure 2: Dependence of the vibration frequency on time and fuel burning-up rate

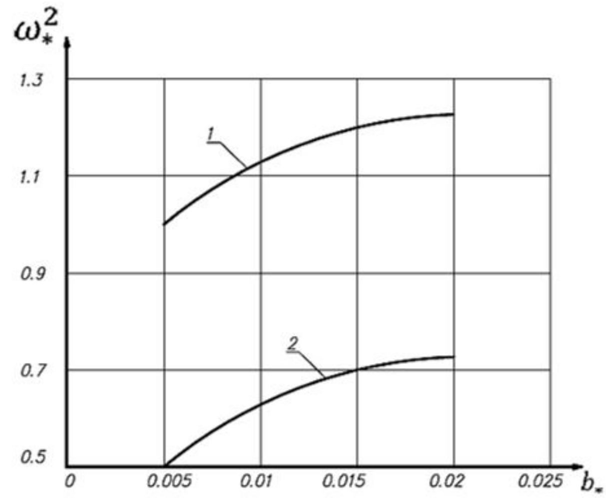

Figure 3: Dependence of the vibration frequency on the stiffness coefficient of the elastic foundation

support. In Fig. 2 value = 0 corresponds to the inoperative engine, and value = 1 – the case when the fuel has completely burned out.

As can be seen from Figures 2-3, with rigid fasting of the ends of the shell, the squares of the fundamental natural frequencies of vibrations are almost two times higher than in the case of their free bearing. Thus, the results obtained make it possible, with a sufficient degree of accuracy, to find the strength dynamic characteristics of the engine cowling at different times of its operation.

CONCLUSIONS

The problem is reduced to a differential equation of motion of the shell, containing discontinuous coefficients with unknowns, the appearance of which is associated with the burnout of the powder charge and, as a result, changes in the zones of action of various loads in the form of temperature fields and internal pressure. The presence of discontinuous coefficients does not allow obtaining a solution to this equation in a closed form, therefore, the approximate Bubnov-Galerkin method is used to solve it. The solution of the static part of the problem is quite traditional, therefore, the study focuses on the problem of dynamics, namely, the determination of the main vibration frequencies of the shell of the solid propellant rocket engine, depending on the degree of burnout of the powder charge. In this case, after applying the Bubnov-Galerkin method, the problem is reduced to a homogeneous system of linear algebraic equations containing the frequency of natural vibrations of the structure as a parameter.

When solving the problem in high approximations, a part of the spectrum of natural vibrations is found from the condition that the determinant of the resolving system of equations is equal to zero. These frequencies can only be found numerically. However, the most important fundamental natural frequency can be obtained in a one-term approximation in a closed-form. It essentially depends on the adopted form of fastening the edges of the shell. It is shown numerically that the rigid fastening of the ends leads to higher values of natural frequencies in comparison with the conditions of free support. Also, the dependence of the fundamental frequency on the degree of burnout of the powder charge (engine operation time) was studied numerically. It is shown that the fundamental frequency decreases with fuel burnup. Also, the dependence of the fundamental frequency on the stiffness coefficient of the powder charge (elastic base) has been numerically investigated. It is also shown numerically that with its increase, the vibration frequency increases. Moreover, all calculations were carried out both for rigid fastening of the ends of the shell, and for their free support.

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